World Transactions on Engineering and Technology Education
Vol.9, No.1, 2011 *© 2011 WIETE*

Conceptual and procedural abilities of engineering students to perform mathematical integration

Chih-Hsien Huang

Ming Chi University of Technology
New Taipei City, Taiwan

ABSTRACT: The main objective of this study was to examine concept-based instruction of definite integrals and its impact on the conceptualisation of integrals by first-year engineering students. Quasi-experimental teaching methods were adopted in this study. Research tools included questionnaires and concept-based teaching models. Subjects for this study were 203 first-year engineering students, who had just finished studying the concept of differentiation. Results of this study revealed that students who received concept-based instruction showed higher levels of comprehension of integration, and the ability to apply learned concepts than students who had received procedure-based teaching.

INTRODUCTION

Results of the Trends in International Mathematics and Science Study (TIMSS) 2003 reveal that Taiwanese students tend to display high achievement, low interest and low self-confidence in mathematics. This indicates that under the pressure to pass examinations, Taiwanese students place too much emphasis on monotonous calculation, which inhibits their developing a sincere interest in mathematics. They fail to establish a systematic and organised network of mathematical knowledge, and lack self-confidence in their own problem-solving abilities.

Traditionally, mathematics teaching and learning were conducted by a process of teacher demonstration, student imitation and exercise practice. An over-emphasis on proficiency in problem solving drove students to focus on calculating correctly, while ignoring the true meaning of the concepts behind the calculations. On the other hand, the impact of a low birth rate and the rapid expansion of universities in Taiwan have led to a decline in the mathematical abilities of university students.

It is for these reasons that enhancing students' understanding of mathematical concepts, as well as their procedural abilities, is an important objective for university mathematics education, and would provide a firm foundation for successful professional development.

LITERATURE REVIEW

Hiebert and Lefevre divided mathematics into conceptual and procedural knowledge [1]. Conceptual knowledge is defined as *knowledge that is rich in connections, which may serve as an interconnected network*. The development of conceptual knowledge relies on the construction of relationships between the fundamental aspects of mathematics and understanding results from the interconnection of knowledge, calculation, and relationships [2].

Procedural knowledge includes two main components; the first is the mathematical symbol representation system, which is the comprehension of mathematical symbols and awareness of symbol syntaxes. The second type consists of the algorithms or rules for solving mathematical tasks. This is the set of directions implemented based on an established sequence of steps. In application, true mathematical understanding has to be constructed based on the connection between these two types of knowledge. As defined by Hiebert and Lefevre, the key concept of integration in calculus includes the definite integral of a function [1].

The Riemann sum was used to derive the relationship between integrals and area, and the Fundamental Theorem of Calculus (FTC). In Orton's research, he observed that most students can apply integration procedures and basic techniques, but have limited understanding of the fundamental concepts of integration [3]. Thompson proposed that *students' difficulties with the integral stem from impoverished concepts of rate of change and from poorly developed and poorly coordinated images of functional covariation and multiplicatively-constructed quantities* [4].

Studies by Thomas and Hong [5] and Belova [6] confirmed that students lack conceptual understanding when learning integration. They can perform simple integral calculations but are unable to solve problems that require conceptual knowledge.

The concept of accumulation, variation and Riemann sums are the key to understanding integration [4]. To understand the many concepts and applications of integration, concepts of accumulation such as arc length, volume and work must be understood. The point of view taken in this study is that the mathematical view of integration as an accumulation function can be regarded as the foundation for teaching integration. The concept of accumulation described in this study was the Riemann sum, and this concept was employed to derive a relationship between definite integrals, indefinite integrals and the Fundamental Theorem of Calculus.

RESEARCH METHODOLOGY

Research participants: For this study, 203 first-year engineering students were chosen, who had just completed courses on differentiation. The students were divided into concept-based and procedure-based groups, which were referred to as the conceptual and procedural groups in this study. The conceptual group was composed of two classes of engineering students with 80 males and 25 females. The procedural group was also composed of two classes of engineering students with 75 males and 23 females. The two groups were taught separately by two teachers. All ten of the class sessions for the groups were recorded on video and class notes were photocopied for analysis of the processes and characteristics of the two kinds of instruction.

Description of conceptual-based teaching model: The procedure-based instruction focused mainly on developing procedural techniques for doing integrals. Most of the class time (almost 80%) was used to introduce integral techniques. Concept-based instruction focused primarily on developing ability to conceptualise integration. Most of the class time (almost 82%) was spent developing the comprehension of concepts. The framework of the conceptual-based instruction module was:

Part A: Antiderivative

1. Using a free fall problem as an example, derive the antiderivative by using the equation $x' = -gt + v_0$.
2. Use the concept of antiderivatives and differential equations, derive the anti-derivatives of exponential functions and logarithmic functions, and anti-differentiation algorithms.

Part B: Riemann sum

1. Accumulation problems of discrete variables and their function values.
2. Accumulation problems of continuous variables and their function values.

Part C: Accumulation and Integration

1. $F(x) = \int f(x)dx$

Where, *f* is the function that describes the variation of *F*, and F is the antiderivative of *f*.

2. $F(x) = \int_a^x f(t)dt$

Where, the value of *F*(*x*) represents the accumulated area under the curve of *f* from *a* to *x*, and the value of *F*(*x*) represents the total change in *F* from *a* to *x*.

Part D: Fundamental Theorem of Calculus (FTC)

(1) $\int_a^b f(x)dx = F(x) - F(a)$ is a unique condition of $\int_a^x f(x)dx = F(x) - F(a)$.

Where, the accumulated area under the curve from *a* to *b* is equal to the total change in *F* from *a* to *b*.

QUESTIONNAIRE

Problem 1:

Calculate the following integrals: a) $\int_0^1 x(x^2+1)^8 dx$ b) $\int_2^3 \frac{x+2}{x-1} dx$

Problem 2:

Using integration, calculate the area of the shaded regions in the following figure.

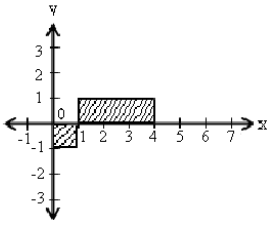

Figure 1: The graph.

Problem 3:

a) Find the area bounded by $f(x) = \sin x$ and the x-axis within the interval $[\pi, 2\pi]$.

b) Compute the integral $\int_\pi^{2\pi} \sin x dx$.

Problem 4:

The graph of $f'(x)$ is given below. The areas of regions A, B, and C are respectively 20, 8, and 5 square units. Find the value of $f(6)$ given that $f(0) = -5$.

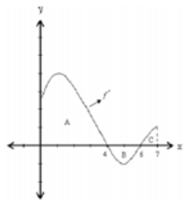

Figure 2: The graph.

Data Analysis

After a period of instruction, the two groups were given 50-minute tests simultaneously. The students' answers were first examined by the researchers together with another senior professor, who confirmed the problem categories. The classification method of Vinner and Dreyfus was used as a reference to categorise the problem-solving processes for exam problems 1 to 4 [7]. Finally, after determining these categories through discussion, the problems were re-categorised, based on the students' problem-solving processes. The students' answers were listed as examples of each category to support the findings of the study.

RESULTS

This section presents the problem-solving results of the two groups of students for the four problems. It examines their conceptual and procedural abilities to perform integration. The processes that the students used to solve Problem 1 can be divided into three categories. Table 1 shows the number and ratio of students in each category. From Table 1, it can be observed that the rate of correct answers for Problem 1a was significantly higher than that for Problem 1b. The reason for this may be that Problem 1a was the standard example used by teachers when teaching integration by substitution. Upon seeing this Problem, students knew how to follow the correct problem-solving procedure.

Conversely, students found it difficult to connect Problem 1b with integration. Viewed from a different teaching perspective, students in the conceptual group performed better than those in the procedural group, with regard to these two problems. These results support the findings of Chappell and Kilpatrick [8], and emphasise that conceptual instruction does not encumber students' proficiency in the integration of procedural techniques:

Category I: Employed correct integration techniques and answered correctly.

Category II: Employed inappropriate integration techniques.

Example: $\int_2^3 \frac{x+2}{x-1}dx = \left.\frac{\frac{1}{2}x^2+2x}{\frac{1}{2}x^2-x}\right|_2^3$

Category III: No answer.

Table 1: Test results of the two groups for Problem 1.

	Category I		Category II		Category III	
	NS	%	NS	%	NS	%
Conceptual Group (Problem1a)	89	84.8	10	9.5	6	5.7
Procedural Group (Problem 1a)	74	75.5	17	17.3	7	7.2
Conceptual Group (Problem 1b)	79	75.2	19	18.1	7	6.7
Procedural Group (Problem 1b)	65	66.3	21	21.4	9	9.3

Note: NS refers to the number of students

The processes used to solve Problem 2 and test results of the students could be divided into five categories. The number and ratio of students are shown in Table 2. From the categories, it can be observed that unclear or ill-formed concepts lead to difficulties in problem solving. In Category II, students did not consider the location of the region when calculating the area by means of integration; they used only simple integral - area concepts. In Category III, even though students considered the location of the region, they were unable to identify the integrand. They believed that the integrand must be in the form of a particular function, for example, $f(x)$, $a(x)$, and x. This may be due to the image that students have about the concept of functions. They did not understand that constant functions could be regarded as an integrand. In Category IV, students had difficulty defining the range of integration and the integrand.

In categories I and III, the proportion of students from the conceptual group who answered correctly was higher than that of students from the procedural group; however, in Category II, the proportion of students from the conceptual group who answered correctly was lower than the proportion from the procedural group.

Category I: Understood the concept of signed area and calculated correctly.

Example: $\int_1^4 1dx - \int_0^1 -1dx = 4$

Category II: Understood the concept of the area but not the concept of signed area.

Example: $\int_1^4 1dx + \int_0^1 -1dx = 3 + -1 = 2$

Category III: Understood the concept of signed area but failed to identify the integrand.

Example: $-\int_0^1 xdx + \int_1^4 xdx$

Category IV: Did not understand the concept of signed area or provided a meaningless integrand.

Example: $\int_{-1}^1 |x|dx$

Category V: No answer.

Table 2: Test results of the two groups for Problem 2 in percentage of correct answers.

	Category I		Category II		Category III		Category IV		Category V	
	NS	%	NS	%	NS	%	NS	%	NS	%
Conceptual Group	73	69.5	8	7.6	18	17.1	5	4.8	1	1
Procedural Group	42	42.9	29	29.6	10	10.2	8	8.3	9	9.2

In problem 3a, the area calculated by the students lay in the third and fourth quadrants. Test results from the students are presented in Table 3. As many as 84.8% of students in the conceptual group were able to answer correctly, while only 26% of those in the procedural group answered correctly. Students who answered incorrectly calculated the area with the same calculation as that of the integration, and did not consider the direction of the curve.

Category I: Understood the concept of signed area and calculated correctly.

Example: $\int_{\pi}^{2\pi} -\sin x dx = \cos x \Big|_{\pi}^{2\pi} = 2$

Category II: Understood the concept of area but not the concept of signed area.

Example: $\int_{\pi}^{2\pi} \sin x dx = -\cos x \Big|_{\pi}^{2\pi} = -2$

Category III: No answer.

Table 3: Test results of the two groups for Problem 3a for percentage of correct answers.

	Category I		Category II		Category III	
	NS	%	NS	%	NS	%
Conceptual Group	89	84.8	10	9.5	6	5.7
Procedural Group	25	25.5	59	60.2	14	14.3

In problem 3b, if students were able to apply the result from Problem 3a, they could immediately obtain the value - 2 for the integration. Test results are shown in Table 4. The objective of Problem 3 was to introduce the relationship between integration and the area. Results showed that students in the procedural group were less able to distinguish between the two concepts than those in the conceptual group. Their concept of integration held that the area was equal to the value of the definite integral. The test performance of the students in the procedural group was similar to the results from Orton's study [3]. When the curve crossed the x-axis, students had difficulty calculating the area, or understanding the relationship between definite integrals and the area below the x-axis.

Category I: Used calculation results from Problem 3a.

Example: The answer to a) is 2, thus $\int_{\pi}^{2\pi} \sin x dx = -2$

Category II: Directly calculated the integral.

Example: $\int_{\pi}^{2\pi} \sin x dx = -\cos x \Big|_{\pi}^{2\pi} = -(\cos 2\pi - \cos \pi) = -2$

Category III: Did not understand the concept of signed area; same answer as a).

Example: $\int_{\pi}^{2\pi} \sin x dx =$ area = 2.

Category IV: No answer.

Table 4: Test results of the two groups for Problem 3b.

	Category I		Category II		Category III		Category IV	
	NS	%	NS	%	NS	%	NS	%
Conceptual Group	62	59.0	31	29.5	10	9.5	2	1.9
Procedural Group	28	28.6	39	39.8	27	27.6	4	4

The problem-solving processes of Problem 4 were divided into five categories. Test results are presented in Table 5. In Categories I and II, students in the conceptual group performed better than those in the procedural group; even though most students in the procedural group were able to apply FTC when solving Problem 1. They did not understand the concept of FTC. They possessed procedural knowledge but not conceptual knowledge of FTC. More than 70% of those in the conceptual group understood the concept of FTC and were able to apply it. About 83% of the students in the conceptual group had an understanding of the sign for area, which was consistent with the results in Problem 3. This indicated that students who had received concept-based instruction had a more in-depth conceptual knowledge of the subject matter.

Category I: Understood the concept of signed area and the concept of FTC.

Example: $\int_0^4 f'(x)dx = f(4) - f(0) = 20$, $-\int_4^6 f'(x)dx = -(f(6) - f(4)) = 8$

Category II: Understood the concept of the signed area, but not the concept of FTC.

Example: $\int_0^4 f'dx - \int_4^6 f'dx + \int_6^7 f'dx = 4 - 0 - 6 + 4 + 7 - 6 = -5$

Category III: Understood the concept of area only.

Example: $\int_0^6 f'(x)dx = 28$

Category IV: Meaningless integration.

Example: $\int_0^7 (ax^3 + bx^2 + cx + d)dx$

Category V: No answer.

Table 5: Test results of the two groups for Problem 4.

	Category I		Category II		Category III		Category IV		Category V	
	NS	%	NS	%	NS	%	NS	%	NS	%
Conceptual Group	75	71.4	12	11.4	8	7.7	6	5.7	4	3.8
Procedural Group	10	10.2	4	4.1	10	10.2	20	20.4	54	55.1

CONCLUSIONS AND EDUCATIONAL IMPLICATIONS

It may be observed while solving problems that students' understanding of the concept of functions has led to difficulties in learning the concept of integration and solving integration problems. In Problem 2, most students in the procedural group did not think that constant function could be regarded as an integrand. In Problem 4, most students in the procedural group did not think that $f'(x)$ could be treated as an integrand. In concept-based classes, lectures on Riemann sums included constant functions (bricklaying problems); exercises using $f'(x)$ as an integrand were included in lectures on indefinite integrals in the procedure-based teaching group. On these two questions, however, students in the conceptual group performed better than those in the procedural group. This demonstrated that for the concept of discrete variables, the results of concept-based instruction were better than those of procedure-based instruction.

From a teaching-oriented perspective, procedure-based instruction about integrals focused on the region of the function graph and the *x*-axis between $[a,b]$, while concept-based instruction focused on the accumulation of the measures of two values. From the accumulation of $f(c)\Delta x$ when $c \in [i\Delta x,(i+1)\Delta x]$, the measure of one value is the function of the other between $[a,b]$. The drawback of the former is that students find it more difficult to expand to other situations. The connection with the area occurs only when $f(c)$ and Δx both represent length, $f(c)\Delta x$ represents a rectangular area with length $f(c)$ and width Δx. Students understood that $f(c)$ and Δx could represent measures of other dimensions (such as force and distance).

Thus, $f(c)\Delta x$ represented the result derived from two measurements (such as work). This indicated that they understood that the formula:

$$\int_a^b f(x)dx$$

was valid in representing the area bounded by the *x*-axis and *f*(*x*), but was not productive. At best, the students only had a superficial understanding of:

$$\int_a^x f(t)dt .$$

In conclusion, the students' performance in this study indicated that in the procedural group, the students' conceptual understanding of the concept of integration was unsatisfactory. The author believes, it is crucial to analyse whether students have sufficient conceptual understanding, and that the concept-based teaching model may greatly benefit the teaching of integration.

ACKNOWLEDGEMENT

This study was supported by the National Science Council of Taiwan under contract number NSC97-2511-S-131-004-MY3.

REFERENCES

1. Hiebert, J. and Lefevre, P., *Procedural and Conceptual knowledge. Conceptual and Procedural Knowledge: The Case of Mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates, 1-27 (1986).
2. Kilpatrick, J., Swafford, J. and Findell, B., *Adding it up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press (2001).
3. Orton, A., Understanding rate of change. *Math. School*, 13, **5**, 23-26 (1984).
4. Thompson, P.W., Images of rate and operational understanding of the Fundamental Theorem of Calculus. *Educational Studies in Mathematics*, 26, **2-3**, 229-274 (1994).
5. Thomas, M.O.J. and Hong, Y.Y., *The Riemann Integral in Calculus: Students' processes and concepts*. In: Clarkson, P.C. (Ed), *Proc. 19th Mathematics Education Research Group of Australasia Conf.*, Melbourne, Australia, 572-579 (1996).
6. Belova, O., Computer Based Approach to Integral Calculus for Prospective Teachers. Unpublished Doctoral Dissertation. Krasnoyarsk State Pedagogical University, Krasnoyarsk, Russia (2006) (in Russian).
7. Vinner, S. and Dreyfus, T., Images and definitions for the concept of a function. *J. for Research in Mathematics Educ.*, 20, 356-366 (1989).
8. Chappell, K.K. and Killpatrick, K., Effects of concept-based instruction on students' conceptual understanding and procedural knowledge of calculus. *Primus*, XIII (1), 17-37 (2003).